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Understanding genetic algorithm dynamics using harvesting strategies

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Abstract

The genetic algorithm (GA) finds optimal solutions over complex fitness landscapes using a method developed in analogy to genetic laws and natural selection. The method essentially operates by optimizing the tradeoff between exploring new points in the search space and exploiting previous information discovered thus far. In this tradeoff, an understanding of the internal GA dynamics, how exactly the GA arrives at an optimum solution, remains somewhat mysterious. Harvesting strategies are introduced here to parameterize the GA's dynamical behavior of elevating sub-threshold solutions toward optimum. The method of harvesting balances the competing aims of population diversity counterweighted against rapid convergence toward the optimum solution. The work establishes that: (1) an upper bound on the fitness ratio exists, above which harvesting becomes too disruptive to the population diversity; (2) analytical conditions for considering elevation within the genetic algorithm are a specific case of logistic growth; and (3) explicit relations exist for the maximum yield and maximum harvestable fraction for 2-stage, 3-stage and finally n-stage harvesting strategies as a function of fitness ratio. Simple expressions for GA time complexity between harvesting steps are presented.

1. Introduction

The genetic algorithm (GA) is capable of optimizing over complex functional landscapes [1–3]. This characteristic develops in part from its ability to consider many solutions in parallel, such that continuity and smoothness no longer become as important to solution processing as in standard hill-climbing methods. The principal challenge, however, remains: how to understand the internal workings of the GA. More specifically, if a step-by-step understanding is found for how genetic pressure pushes solutions towards optimum, then various improvements and accelerations should quickly follow.

In this effort, an n-stage harvesting strategy is examined here for a genetic algorithm which promotes most fit individuals without further cross-over or mutation. Within each harvesting stage and also within each generational time step, the less fit individuals are allowed to cross-over. The overall product is a time-varying elite group which is not disturbed genetically, coupled with a subthreshold population

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which evolves according to standard genetic laws. The entire process balances two competing aims: the GA both encourages diversity among sub-optimal performers, while at the same time preserving the integrity of better performers.

Within this procedure, the class of performance is set by assigning an excursion level [4] (or "water level") which floats selected solutions above low-lying peaks on a complex fitness landscape. The behavior of such a two-tiered genetic algorithm (solutions partitioned above and below an excursion level) has been outlined elsewhere [4]; in general we call the newer method, the excursion set mediated genetic algorithm, or the ESMGA (Fig. 1). It has produced improved properties for convergence on selected test functions, both experimentally on landscapes with multiple optima and analytically as a candidate for seeking a stronger GA version using Holland's schema theorem. A more complete characterization is in progress.

This paper develops a general harvesting strategy for supporting the elevation of sub-threshold solutions above the excursion level. First, the dynamics of a generational genetic algorithm are presented, wherein all individuals below the threshold are replaced at each generation. For this generational GA, population dynamics are shown to follow as a specific case of logistic growth. Harvesting is considered as a stronger force for achieving rapid optimizations for the cases of 2-stage harvesting, 3-stage harvesting and finally n-stage harvesting. Criteria for answering the important practical questions (of when to harvest) are formulated in terms of the fitness ratio between binary

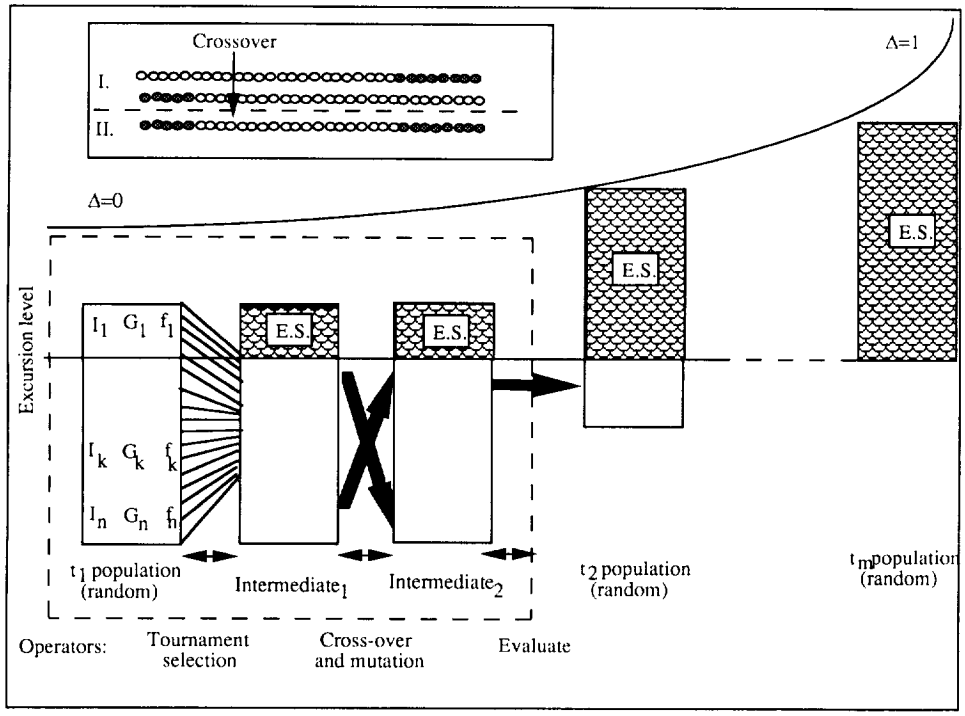

Fig. 1. Schema showing modified GA model (ESMGA) and corresponding harvesting strategy for intergenerational promotion. The dashed box encloses the harvested fraction of the entire population, with cross-over and mutation performed on the less-fit members lying below the excursion level. At the conclusion of a GA run, the entire remaining population (above and below threshold) are combined for evaluating the total yield.

choices. In the last section of the paper, we propose recommendations for driving the GA more strongly using the harvesting approach.

2. Population dynamics of generational reproduction and logistic growth

Consider a genetic algorithm which replaces all individuals at each generation, the so-called generational GA [5,6]. Its dynamic behavior contrasts markedly with a steady-state model which allows birth and death of individuals to keep the population level constant. With generational reproduction, the population level is not fixed and new individuals can be allowed to enter the solution space. For an initial population, P_0, which is allowed to reproduce over a time sequence, t, and which is evaluated over a fitness landscape characterized by the fitness ratio, $r = f_1/f_0 > 1$, then the final population, P_t is

$$P_t = \frac{r^t P_0}{1 + P_0(r^t - 1)} = \frac{\lambda P_0}{1 + P_0(\lambda - 1)}, \tag{1}$$

where $\lambda = r^t$.

This form (1) of the generational GA can be made to correspond to a more general logistic equation, $dP/dt = (a - bP)P$ for the case of $a = 1, b = 1, r = e = 2.732\ldots$, where the fitness ratio is simply taken as the exponential. One motivation for establishing this link between the generational GA and logistic behavior is both to take advantage of (and to make contact with) the vast literature on this aspect of biological population dynamics.

The modified ESMGA protocol (Fig. 1) can be represented within the generational model of the standard GA by the following pseudo-code:

```
procedure ESMGA
    begin
        t = 0;
            initialize n(t);
            evaluate structures in n(t) and identify fitnesses
                     greater than the excursion level, n_ε(t);
            while termination condition not satisfied
                     (convergence parameter Δ < 1) do
        begin
            t = t + 1;
            select n(t) from n(t − 1) using excursion set
                       mediation and tournament solution;
            modify structures in n(t)-n_ε(t) by applying genetic operators;
            evaluate structures in n(t)
        end
    end.
```

The thrust of the modified code is to set aside a time-varying elite population (defined for each generation as those members with fitnesses evaluated above the excursion level) and not mix these high performers with lower-scoring members (Fig. 1). Most notably this formalism has been shown to

provide a basis for insuring a stronger form [4] of Holland's schema theorem, in which individuals converge towards the optima with an exponential approach. Simulations and demonstrated efficacy of the ESMGA appear most explicitly in Ref. [4]. A natural corollary to this notion of excursion levels is harvesting. In the following sections, we adopt various strategies [7,8] for harvesting high-fitness GA solutions and evaluate their characteristic population dynamics using Eq. (1). Harvesting can in turn be considered a kind of promotion for lower-fitness solutions; at the completion of a GA run, the entire population is promoted (or culled) and a maximum obtainable fraction of the initial population is evaluated.

3. Harvesting in 2 stages: when to harvest

As an initial approach to achieving improved promotion dynamics of subthreshold solutions, we examine a 2-stage harvest. The harvesting sequence is shown schematically in Fig. 2a. Let the initial population commence (below the excursion level, ϵ) with P_0 at time t_0 and subsequently grow through generational reproduction (through point-mutation, cross-over and selection) to an intermediate population, P_1 at time t_1. From this intermediate population, we will harvest or cull a fraction k $(0 < k < 1)$ of P_1, so that the new initial population for the second stage is $(1 - k)P_1$. For this two-stage harvest, the total yield of solutions, H, promoted above the excursion level, can be written as the sum of two terms, the first corresponding to standard generational growth within the GA. The second term corresponds to a modified growth which depends on three factors: the fitness ratio, the number of generations allowed until harvesting and the initial population.

We imagine the 2-step growth cycle as pictured (Fig. 2a), with k parameterizing the harvest fraction.

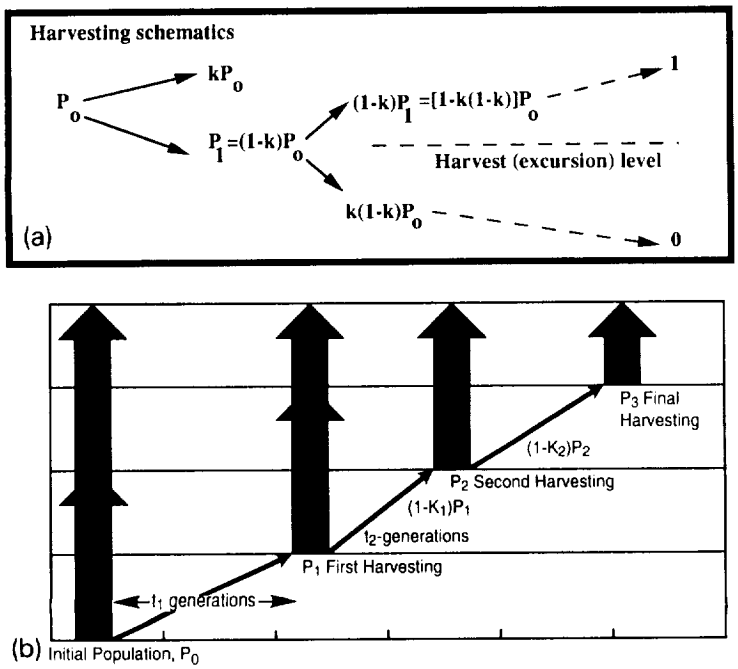

Fig. 2. Population dynamics within 2-stage (a) and 3-stage (b) harvesting models.

We ask the question: what harvest fraction gives the maximum number of solutions promoted above the excursion level? Algebraically, the problem is to maximize P as the k value that gives $\partial H/\partial k = 0$. First we solve for the harvested population H:

$$H = k_1 P_1 + P_2 = k_1 P_1 + \frac{(1 - k_1) P_1 \lambda}{1 + (1 - k_1)(\lambda - 1) P_1}, \tag{2}$$

where the second equality follows from Eq. (1) written as $P_t = P_2$.

In terms of the fitness ratio and generation time (written as $\lambda = r^t$), the harvest fraction which maximizes promotion above the excursion level is

$$k = 1 - \frac{1}{(\lambda^{t/2} + 1) P_1} = 1 - \frac{1}{(r^{t/2} + 1) P_1}. \tag{3}$$

The optimal harvest fraction k is shown graphically in Fig. 3. Clearly, the depth into the subthreshold population that the harvest can be culled depends on the fitness ratio, r, the number of generations until harvest, t, and initial population size, P_0. Harvest depth increases for high fitness ratios, $r \gg 1$, for long generation times, t ($t \gg 1$) and for large initial population sizes, P_0.

When (3) is substituted back into (2) for $H_{\max}$, then

$$H_{\max} = P_1 + \frac{(r^{t/2} - 1)}{(r^{t/2} + 1)}. \tag{4}$$

Fig. 4 shows the maximum promotable fraction of the population that can be achieved within a 2-harvest arrangement.

A deeper level of understanding can be gained about harvest dynamics from consideration of Eq. (3) for the harvest fraction, k. In particular, since harvest benefits the overall yield only for the condition $0 < k < 1$, a minimum condition on the fitness ratio can select problems which most probably would benefit from culling. In this spirit, we ask the question for a 2-harvest cycle: when does harvesting benefit the GA approach to superthreshold values?

To answer this question, we propose $k > 0$ as the evaluation and solve for fitness ratio in terms of generation time and population size, P_0. From Eq. (3), $k > 0$ defines a critical fitness ratio, r_{crit} which limits those problems which harvesting benefits to fitness ratios greater than

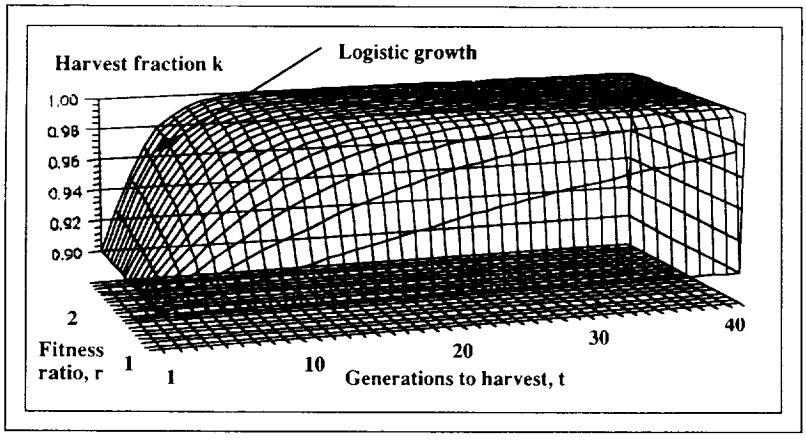

Fig. 3. 2-stage harvestable fraction, k, as a function of generation time and the fitness ratio, r.

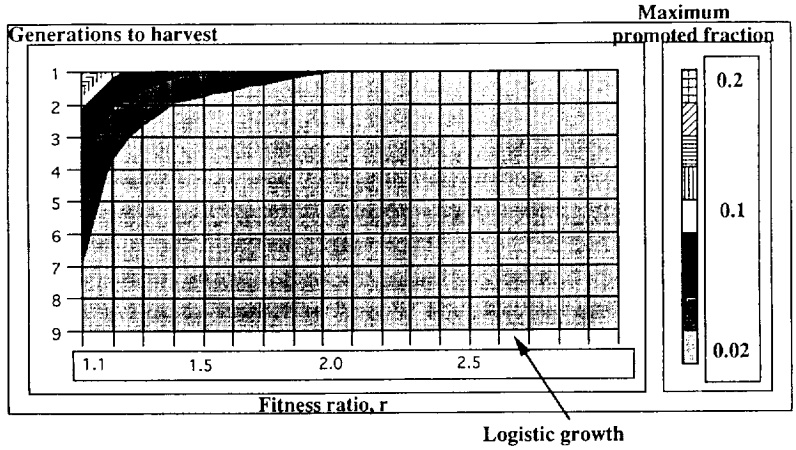

Fig. 4. Maximum yield shown as a function of generation time and the fitness ratio, r.

$$r > r_{\text{crit}} = (1 + 1/P_0)^{2/3t} . \tag{5}$$

The harvest fraction is a monotonically increasing function of the fitness ratio; higher fitness ratios will naturally lead to a deeper fraction of culling. For the minimum condition (5) therefore, the minimum fitness ratio is r_{crit}, below which harvesting itself cannot profit the simulation. Thus for problems with low fitness ratios, harvesting is advisable only for short generation times (t small) or for few strings (P_0 small). Conversely, one should not harvest a GA population prematurely if $r < r_{\text{crit}}$. In some sense, this result captures the essential dilemma of GA dynamics: how to span a large population space (maintain genetic diversity) while at the same time converging onto an optimal solution. Harvesting offers one bridge towards understanding more deeply this tradeoff.

To conclude this analysis, we demonstrate the independence of the last harvest cycle from initial conditions on population size. Dynamically, this can be seen as a final catch of all individuals which depends only on the fitness ratio, r and the generation time, t. Combining Eqs. (3) and (4), and solving for P_2, then

$$P_2 = \frac{\lambda^{t/2}}{(\lambda^{t/2} + 1)} = \frac{r^{t/2}}{(r^{t/2} + 1)} . \tag{6}$$

To summarize this section, a two-stage harvest can be seen to: (1) establish harvest criteria (in what types of fitness landscapes is harvesting favored); (2) link the generational GA dynamics to the well-developed study of logistic growth for the special case of unit growth rates for birth ($a = 1$) and death ($b = 1$) and a particular fitness ratio, $r = e = 2.732$. In other words, on high fitness landscapes, logistic growth can well describe generational GA. This latter identification opens up a more complete contact between GA dynamics and classical models for biological growth. In addition, the 2-stage model is simple enough to yield analytical solutions (4), (5) for the maximum promotable population in a subthreshold community, $H_{\max}$, as well as giving the particular optimum harvest fraction, k, which achieves this strong pressure.

4. Harvesting in 3 stages: in what proportion to harvest

The 3-stage harvesting sequence is shown schematically in Fig. 2b. Let the initial population commence (below the excursion level, ε) with P_0 at time t_0 and subsequently grow through generational reproduction to an intermediate population, P_1 at time t_1. From this intermediate population, we will again harvest or cull a fraction k_1 $(0 < k_1 < 1)$ of P_1, so that the new initial population for the second stage is $(1 - k_1)P_1$. The cycle of generational growth and subsequent harvest is repeated for a second (independent) harvest fraction, k_2. For this three-stage harvest, the total yield of solutions, P, promoted above the excursion level, can be written as the sum of three terms, the first corresponding to standard generational growth within the GA, the second and third again corresponding to a modified growth dependent on the fitness ratio, generations allowed until harvesting and the initial population. In the three-stage process, we pay particular attention to the mixture of possible scenarios based on different harvest fractions. The simple cases of an increasingly deep cut, a constant cut, and a decreasing cut from the subthreshold population will be treated; the increasing cut will be discussed for its relevance towards driving the subthreshold population to extinction, namely to arrive at a total promotion rate above the excursion level.

We imagine the 3-step growth cycle as pictured (Fig. 2a), with the set (k_1, k_2) parameterizing the harvest fractions taken from 2 intermediate populations. We again ask the question, what harvest fraction gives the maximum number of solutions promoted above the excursion level? Algebraically, the problem is to maximize Π as the k value that gives $\partial \Pi / \partial k_1 = \partial \Pi / \partial k_2 = 0$. First we solve for the harvested population Π:

$$\Pi = k_1 P_1 + P_3 + k_2 P_2 = k_1 P_1 + \Phi(k_1, k_2) + \beta(k_1, k_2)\,, \tag{7}$$

where the last two terms in Eq. (7) can be written for $P_t = P_1$ as

$$\Phi(k_1, k_2) = k_2 P_2 = \frac{k_2 \lambda}{1 + P_1(1 - k_1)(\lambda - 1)} (1 - k_1) P_1 \tag{8a}$$

and

$$\begin{aligned} \beta(k_1, k_2) = P_3 &= \frac{\lambda}{1 + P_2(1 - k_2)(\lambda - 1)} (1 - k_2) P_2 \\ &= \frac{\lambda^2 (1 - k_1)(1 - k_2) P_1}{1 + (\lambda - 1)(1 - k_1) P_1 + P_1 \lambda (\lambda - 1)(1 - k_1)(1 - k_2)}\,. \end{aligned} \tag{8b}$$

In terms of the fitness ratio and generation time (written as $\lambda = r^t$), then the harvest fractions, (k_1, k_2), which maximize promotion above the excursion level are:

$$k_1 = 1 - \frac{1}{(\lambda^{1/2} + 1) P_1} = 1 - \frac{1}{(r^{t/2} + 1) P_1}\,, \quad k_2 = 1 - \frac{1}{r^{t/2}}\,. \tag{9, 10}$$

The optimal harvest fractions k_i are shown graphically in Fig. 5. While the optimum harvest fraction from the first cycle (Eq. 9) is the same symbolically as for the 2-stage problem (4), the numerical value is diminished since the total generation time $T = t/3$, instead of $T = t/2$. Clearly, the depth into the subthreshold population that the harvest can be culled depends on the fitness ratio, r, and the number of generations until harvest, t, but the second harvest fraction *does not depend on the initial population size*, P_0. As was the case previously, harvest depth increases for high fitness ratios, $r \gg 1$, for long

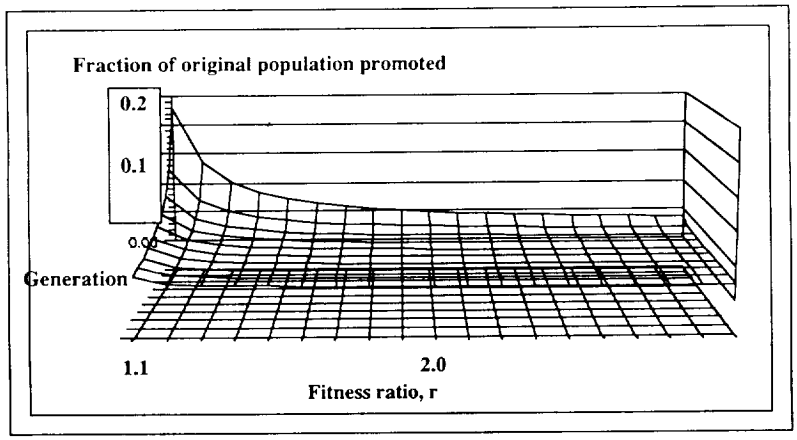

Fig. 5. 3-stage harvestable fraction, k, as a function of generation time and the fitness ratio, r.

generation times, t, and for large initial population sizes, P_0. In general the second harvestable fraction, k_2 is less than or equal to k_1 and increases steeply as harvest time is delayed until later generations.

When (9,10) are substituted back into (8) for $H_{\max}$, then

$$H_{\max} = P_1 + \frac{(r^{t/2} - 1)}{(r^{t/2} + 1)} 2 > H_{\max}(\text{2-stage harvest}) . \tag{11}$$

Eq. (11) shows the maximum promotable fraction of the population that can be achieved within a 3-harvest arrangement compared to a 2-stage harvest.

Further understanding can be gained about harvest dynamics from consideration of Eqs. (9, 10) for the dual harvest fractions, k_i. In particular, since 3-stage harvesting benefits the overall yield only for the condition when both fractions $0 < k_i < 1$ then an additional minimum condition on the fitness ratio (10) can select problems which most probably would benefit from 3-staged compared to 2-staged culling. In this spirit, we ask the question for a 3-harvest cycle: how much harvesting benefits the GA's approach to super-threshold values?

To answer this question, we propose both $k_i > 0$ as the evaluation and solve for fitness ratio in terms of generation time and population size, P_0. From Eq. (8)–(10), $k_{1,2} > 0$ defines a critical fitness ratio, r_{crit}, which limits those problems which harvesting benefits to fitness ratios greater than

$$r > r_{\text{crit}} = \left(1 + \frac{1}{P_0}\right)^{2/3t} , \quad r > r_{\text{crit}} = 1 , \tag{12}$$

where the second inequality follows for all fitness ratios (by definition of the rank ordering of binary fitnesses, $r = f_1/f_0 > 1$). As before (5) for problems with low fitness ratios, Eq. (12) indicates that harvesting is advisable only for short generation times (t small) or for few strings (P_0 small). Conversely, one should not harvest a GA population prematurely if $r < r_{\text{crit}}$.

To summarize this section, a three-stage harvest can be seen to: (1) establish additional harvest criteria (in what fitness landscape is harvesting beneficial); (2) introduce three possible scenarios (Fig. 6) for the harvesting envelope and its effect on population diversity. The three scenarios include increasingly deep harvesting fractions (strong pressure, $k_1 < k_2$), constant harvesting fractions (constant pressure, $k_1 = k_2$), and decreasingly deep harvesting fractions (light pressure, $k_1 > k_2$). For an

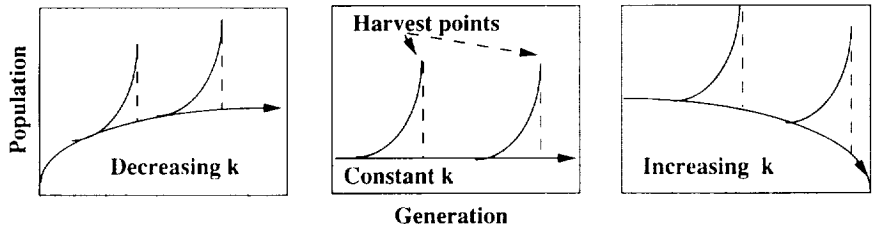

Fig. 6. Case studies of scenarios for depth of harvesting fraction (light vs. heavy pressure for a culled fraction).

increasingly deep harvesting fraction, the extinction or depletion of the sub-threshold population is possible. In short, this case corresponds to depleting the diversity, such that not enough sub-threshold reproduction takes place. (In practice, for very few strings, the assumptions required for treating the population growth as a differential break down.) In an important sense, the case of decreasingly deep harvesting fractions corresponds schematically to the previous description of ESMGA. In other work, the case of constant harvesting, $P_{t+1}/P_t = 1$, has been generalized to include linear harvesting profiles, $P_{t+1}/P_t = \text{constant}$. Several interesting analytical results can be derived in closed form for such linear harvesting pressure.

Finally, as was the case for 2 stages, the 3-stage model remains simple enough to yield analytical solutions for the maximum promotable population in a subthreshold community, $P_{\max}$, as well as giving the particular optimum harvest fractions, k_i, which achieves this strong pressure.

5. Harvesting in n stages: how to harvest continuously

The n-stage harvesting cycle, or how to harvest continuously from sub-threshold populations, is a straightforward extension of the 3-stage results. The optimal harvesting fractions k_i follow from the same condition on the maximum yield, $\Pi_{\max}(\partial \Pi/\partial k_i = 0)$:

$$k_1 = 1 - \frac{1}{(\lambda^{1/2}+1)P_1} = 1 - \frac{1}{(r^{t/2}+1)P_1}, \tag{13}$$

$$k_2 = 1 - \frac{1}{r^{t/2}} = k_3 = \cdots = k_n . \tag{14}$$

The n-stage generalization of the later harvesting populations similarly follows as an extension of (11) as

$$P_2 = \frac{\lambda^{t/2}}{(\lambda^{t/2}+1)} = \frac{r^{t/2}}{(r^{t/2}+1)} = P_3 = \cdots = P_n . \tag{15}$$

Finally the maximum yield is

$$\Pi_{\max} = P_1 + \frac{(r^{t/2}-1)}{(r^{t/2}+1)}(n-1) > \Pi_{\max}(\text{limit of fewer than } n \text{ stages}) . \tag{16}$$

Eq. (16) can be interpreted as the maximum promotion of subthreshold individuals *under continuous harvesting*. An important result familiar from logistic growth is the limit of maximum promotion under the ESMGA as

$$P_{\max} = P_1 + T \quad \text{for } n(\text{large}) \gg 1 \,, \tag{17}$$

where T is the total number of generations divided into equal harvest lifetimes, $T = nt$. This last result gives an upper-bound estimate for the ESMGA's promotion capability (how much lift can be given to sub-threshold populations). Its fundamental significance derives from its analytical simplicity and dependence only on the GA's assignable or free initial conditions, P_0, T. Both the initial population size, P_0, and the total number of generations, T, can typically be fine-tuned at the outset by the operator, such that the harvesting scenario can suitably be predicted prior to a given GA run. For the n-stage limit, this last finding (17) represents the primary result of this section.

The n-stage harvesting limit was tested experimentally using the ESMGA. Two simulations are included and analyzed on a badly scaled function and a roughly peaked Riccati polynomial. Results are tabulated for harvest fractions in a $T = 15$-stage harvesting season. The total number of generations ($T = 15$) is divided into five equal cycles ($n = 5$) of three generations each ($t = 3$). Rapid convergence onto the global optima is reported in Fig. 7.

It is important to note that, although the analytical forms (13)–(16) are similar for the 2-, 3- and n-stage harvests, the actual numerical convergence behavior will differ dramatically. Notably a different time interval, nt, is considered in each case and because the generation time appears in the exponential, $\lambda = r^t$, then the individual population dynamics will vary significantly between different harvest scenarios. Further it is good to emphasize that the yield from one harvest cycle to the next need not necessarily increase, $P_n > P_{n-1}$, since in general stronger or lighter pressure can be applied freely between cycles within any given harvesting method.

6. Generalized GA dynamics

6.1. Generational vs. steady-state reproduction within the ESMGA

An appealing feature of introducing threshold levels into the GA is a renewed capacity to span the two principal types of reproductive strategies. For example, the ESMGA combines somewhat contrasting techniques of generational and steady-state reproduction into a single simulation. For the most fit population, the reproduction is closest to steady state. During GA evolution, the proportional change in population approaches a constant. This effect corresponds to light pressure on superior performers. For sub-threshold populations, on the other hand, the dynamics most strongly follows a generational model. The population number changes much more dramatically, a case which corresponds to stronger pressure on sub-threshold populations. Thus by pressuring less fit individuals, overall performance is optimized while at the same time conserving more fit levels. The conservation strategy in particular maintains population strength without excessive disruption. The ESMGA can now be considered a composite model which captures the best features of generational and steady-state reproduction into one algorithm.

6.2. Time complexity within harvesting steps

The time complexity of a GA refers to the functional dependence of convergence time on the population size and fitness [9,10]. Two cases of time complexity can be considered: worst and average limits. The *worst case complexity* corresponds to the conditions ($P_f = 1 - (1/n)$; $P_0 = 1/n$) where $1/n$ is

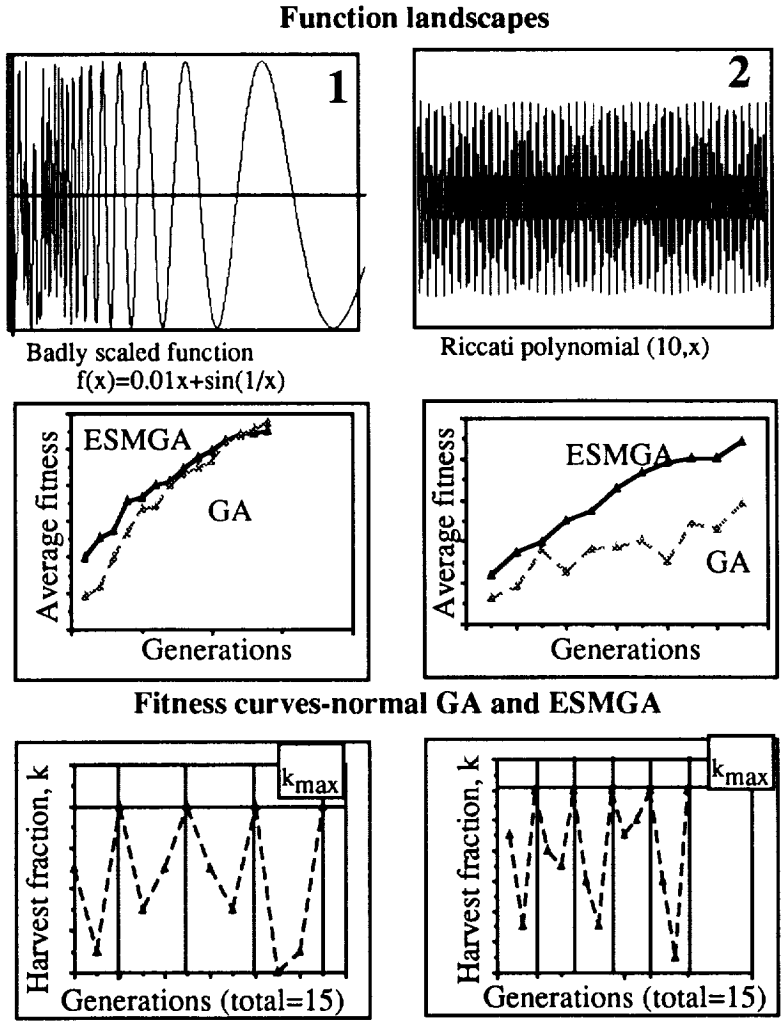

Fig. 7. Test functions and ESMGA performance. Results shown for a badly scaled function $(0.01x + \sin(1/x))$ and the Ricatti polynomial $\{10, x\}$. Upper two windows show the function landscape; middle two windows compare fitness curves evaluated for normal GA and ESMGA operation. The ESMGA is run with five excursion ("water") levels normalized between 0 and 20 as $\varepsilon = \{4.5, 7.5, 10.5, 13.5, 16.5\}$. The bottom two windows show the evolution of the harvest fraction for both functions over time.

some small number equal to the proportional representation of a fit individual in a general population either initially P_0 or later (near convergence) at the end of a harvesting step, P_f. For the average case complexity, on the other hand, the fit individual in a binary scheme is initially represented with probability $P_0 = 0.5$ (and not equal to $1/n$). Analytically, these worst and average cases for convergence can be found for the pre-harvest population as $P_1 = P_f = 1 - (1/n)$, $P_0 = 1/n$ substituted into (4) and solved explicitly for generation time t as

$$t_c = \frac{\ln(n-1)}{\ln r} \qquad \text{(average)}\,, \tag{18}$$

$$t_c = \frac{2\ln(n-1)}{\ln r} \quad \text{(worst)} \tag{19}$$

for the first harvest stage. For all subsequent n-stages the time complexity is the same for both worst and average cases:

$$t_c = \frac{2\ln[(2n-1)/n]}{\ln r} \quad \text{(average and worst)}\,. \tag{20}$$

Result (20) for n-stage harvesting is particularly noteworthy, since for a large number of harvesting steps ($n \gg 1$), the time complexity becomes independent of the population size, n. In this way, subsequent stages of harvest differ in their time complexity compared to the first pre-harvest stage (18), (19).

6.3. *Harvesting recommendations*

One advantage of the staged harvesting approach is how each stage contributes an additional layer to possible GA scenarios. For example, given a 2-stage harvesting cycle, an analytical criterion (5) exists for finding promising fitness landscapes (or ratios) which might benefit from harvesting, the so-called *harvesting criterion*. For 3-stage harvesting, the concept of *an extinction solution* appears, such that increasingly deeper harvesting fractions deplete sub-threshold solutions. Finally for n-stage harvesting, the continuous limit gives a maximum yield of superior performers which does not depend on the shape of the fitness landscape (as measured by the fitness ratio) at all. Rather for n stages, there exists *a fitness-free solution* (17). In this last case, initial parameters of population size and generation times alone establish the yield on any arbitrary landscape. Thus the continuous harvesting limit can improve yields over landscapes with any steepness of fitness values.

6.4. *Towards an ESMGA Philosophy*

The power of the ESMGA derives from its balanced dynamics which combine features of both generational and steady-state reproduction. For sub-fit populations, generational dynamics dominate since the population size varies considerably (owing to continuous promotion from a limited and diminishing pool), and also since all individuals are replaced and crossed over at each generational time step. For super-fit populations, on the other hand, the converse dynamics of steady state dominates, since convergence must necessarily decrease the relative importance of freshly inputted solutions. In the limit, the population dynamics above excursion level should approximate steady state. The success of excursion sets therefore derives from its flexible dynamics which fuse alternative reproductive strategies.

The question of how to set the excursion level is fundamental. If this threshold is taken to describe a water level which submerges sub-fit solutions on rugged (fitness) landscapes, then the ESMGA dynamics become more clear. In particular, the ESMGA must work by identifying, then clustering, these local maxima which jut out above the water level. Only following this initial clustering does global selection begin as a more limited search.

We propose several research problems for future work. Clearly within ESMGA formalism, some notion of an adaptive time step should be implemented. Standard forms would slowly shrink the time step as convergence approaches. Higher resolution of closely matched optima could then emerge in the

limit of no promotion and a global solution. A priority task therefore should identify functions with special suitability for ESMGA solution. An example might present various ill-behaved functions (fractal landscapes, discontinuities of differing severity, time varying functions, etc.) and quantify the functional complexity which confounds the ESMGA search. In this field of deceptive GA functions, the ESMGA may provide the necessary genetic pressure for confidently searching and selecting.

7. Concluding remarks

In summary, a harvesting strategy adapted to examine GA dynamics can

(1) transparently connect the generational GA to standard logistic growth models;

(2) establish analytical criteria for answering the practical question of when to harvest and what fitness landscapes are likely to benefit from harvesting;

(3) identify the maximum harvestable population (for promotion above some fitness threshold);

(4) solve for the fraction of subthreshold individuals for whom promotion proves beneficial.

An additional outcome from the harvesting framework is proof that the last harvesting cycle does not depend on the initial conditions except through the fitness ratio itself. Future work will introduce the number of harvesting stages as a variable parameter which can be adjusted to pressure the best individual's fitness continuously. Taking advantage of this cycle of initial relaxation, followed by additional pressure, preliminary experiments performed on sample functions have shown rapid convergence to near global optima. The ongoing work should bear on convergence properties of any GA, in that the approach to optimum involves harvesting strategies in standard versions of elite selection, niche formation or sharing functions. Selection itself, the central component of GA operation, should operate analytically as a kind of harvesting, not only if successively applied across generations, but more importantly as it appears *between generations* of any genetic algorithm dynamic. The final aim is to push GA dynamics to its performance limit, while establishing a workable framework for understanding its internal methods for arriving at that optimum performance through harvesting.

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